

Quasi-Long-Range Order in the Calogero-Sutherland Model

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The occurrence of quasi-long-range positional order in the ground-state of the one-dimensional repulsive Calogero-Sutherland model is studied. By mapping the exact ground-state into a one dimensional classical system of interacting particles at finite temperatures the structure function and the displacement correlation functions are calculated numerically using Monte Carlo simulation methods. These are found to exhibit quasi-long-range positional order for all values of the parameters. The exponent characterizing the algebraic decay of the displacement correlation functions with distance is estimated. It is argued that the ground-state of the repulsive Calogero-Sutherland model consists of a single normal phase with quasi-long-range positional order.

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I. INTRODUCTION

It is well established that in a classical two-dimensional (2D) system of particles, interacting through a potential that falls off sufficiently fast with distance, long-range positional order cannot exist at finite temperatures [1]. Instead, a low temperature phase with quasi-long-range positional order (QLRPO) occurs. This phase is characterized by particle displacement correlation functions that decay algebraically with distance, with a temperature-dependent exponent. This leads to a structure function with power law singularities at some reciprocal lattice vectors. It is predicted that this phase is destroyed, at a temperature T_c , by a phase transition that can be either first order or continuous. The latter resulting from the unbiding of topological excitations. It is also known that theoretical considerations that take into account only thermal phonon fluctuations in the harmonic approximation correctly predict the low temperature properties of this phase [2]. These predictions are confirmed by Monte Carlo simulations of model systems [3].

For a quantum system of bosons in one-dimension (1D), interacting through a potential that falls off sufficiently fast at large distances, long-range positional order is also ruled out in the ground-state. For this system, theoretical analysis that accounts for zero-point phonon fluctuations in the harmonic approximation make predictions analogous to those for classical 2D systems: destruction of long-range positional order, power-law decay with distance of displacement correlation functions and power-law singularities in the structure function at some reciprocal lattice vectors. However, QLRPO in the ground-state of such 1D bosons is not as well established as in classical 2D system,

In this paper we study in detail positional order in

a 1D system of bosons interacting through a repulsive potential that varies inversely with the square of the distance - the Calogero-Sutherland (CS) model - for which the exact ground-state wavefunction is known [4]. The structure function for the CS model was calculated in the harmonic approximation by several authors [5]. The results confirm that it has power law singularities at reciprocal lattice vectors with an exponent that varies continuously with the model parameters. However, the range of validity of this approximation it is not known. It is unclear whether QLRPO persists outside this range and, if so, what is the exponent governing the power-law decay of the displacement correlations. We address these questions here.

In order to go beyond the harmonic approximation we resort to numerical calculations. Sutherland obtained several years ago the exact ground-state wavefunction for the CS model [4]. Based on this wavefunction we show that the ground-state of the CS model can be mapped into a classical 1D system of interacting particles at finite temperatures. By applying Monte Carlo (MC) simulation methods [6] to this classical system we calculate the structure function and investigate the occurrence of QLRPO for arbitrary values of the model parameters.

We show evidence that the CS model ground-state has a single phase with QLRPO and argue that this phase is non-superfluid. Over a range of parameter values we find that QLRPO leads to power law singularities in the structure function and estimate the exponents governing these singularities by finite size scaling analysis of the MC data. Outside this range we find that the structure function has no singularity, and that the displacement correlation functions decay algebraically with distance. We also estimate the exponent governing this decay. We find that this exponent differs from that predicted by the

harmonic approximation only in the region where quantum fluctuations are large, being smaller than the harmonic ones there.

For particular values of the CS model parameters the structure function has been calculated exactly by Sutherland [4]. Our numerical results are found to be in good agreement with the exact ones.

This paper is organized as follows. In Sec.II we review the exact results obtained by Sutherland and derive the mapping into a classical 1D interacting system. In Sec.III we review the harmonic approximation for the CS model and show that Sutherland's exact ground-state wavefunction reduces to the harmonic one in the semiclassical region. In Sec.IV we explain our numerical method in detail and report the results obtained by it. In Sec.V we interpret these results and state our conclusions.

II. GROUND-STATE WAVEFUNCTION

The CS model describes N bosons of mass m on a line of length L interacting through the two-body potential

$$V(r) = \gamma \sum_{n=-\infty}^{\infty} (r + nL)^{-2} = \frac{\gamma\pi^2}{L^2} \left[\sin \frac{\pi r}{L} \right]^{-2}, \quad (1)$$

where γ is a constant that we assume positive. This potential is periodic, with period L . For large L the $n = 0$ term in Eq. (1) is the most important, so that $V(r)$ varies essentially as r^{-2} [4].

Sutherland found that the exact ground-state wavefunction for this system is given by

$$\Psi = \text{const.} \prod_{i>j} \left| \sin \frac{\pi(x_i - x_j)}{L} \right|^\lambda, \quad (2)$$

where

$$2\lambda \equiv \beta = 1 + (1 + 2g)^{1/2}. \quad (3)$$

The dimensionless parameter $g = \frac{4m\gamma}{\hbar^2}$ measures the relative strengths of the potential and kinetic energies. Sutherland also found that the exact ground-state energy is given by

$$E = N \frac{\hbar^2}{2ma^2} \frac{\pi^2 \beta^2}{12}, \quad (4)$$

where $a = L/N$ is the mean interparticle distance.

The ground-state average of any operator that depends only on the position operators x_i ($i = 1, 2, \dots, N$), $A(\{x\})$, is given by

$$\langle A \rangle = \frac{\prod_{i=1}^N \int_0^L dx_i A(\{x\}) |\Psi|^2}{\prod_{i=1}^N \int_0^L dx_i |\Psi|^2}. \quad (5)$$

Using Eqs. (2) and (3) $|\Psi|^2$ can be written as

$$|\Psi|^2 = \exp \left\{ -\beta \frac{1}{2} \sum_{i \neq j} \mathcal{V}(x_i - x_j) \right\}, \quad (6)$$

where

$$\mathcal{V}(x) = -\ln \left| \sin \frac{\pi x}{L} \right|. \quad (7)$$

Thus,

$$\langle A \rangle = \frac{\prod_{i=1}^N \int_0^L dx_i A(\{x\}) \exp \left\{ -\beta \frac{1}{2} \sum_{i \neq j} \mathcal{V}(x_i - x_j) \right\}}{\prod_{i=1}^N \int_0^L dx_i \exp \left\{ -\beta \frac{1}{2} \sum_{i \neq j} \mathcal{V}(x_i - x_j) \right\}}, \quad (8)$$

According to this equation $\langle A \rangle$ may be calculated as the canonical ensemble average of $A(\{x\})$ of a 1D classical system of particles interacting through the fictitious two-body potential $\mathcal{V}(x)$. The parameter β plays the role of the inverse temperature.

The numerical calculations carried out in this paper are based on Eq. (8), as discussed in Sec.IV

III. HARMONIC APPROXIMATION

Here we review the harmonic approximation for the 1D boson system described in Sec.II.

A. Phonon Spectrum and Wavefunction

The interaction energy of the 1D boson system under consideration here is

$$U = \frac{1}{2} \sum_{i \neq j} V(x_i - x_j). \quad (9)$$

As usual, the harmonic approximation consists in expanding U to second order in the displacements, u_n , from the classical equilibrium positions $X_n = na$ [7]. The result is

$$U = \frac{1}{2} \sum_{n \neq m} V[X_n - X_m - (u_n - u_m)] \equiv E_{cl} + \delta U, \quad (10)$$

where E_{cl} is the classical ground-state energy and δU is the harmonic correction

$$\delta U = \frac{1}{2} \sum_{n \neq m} \frac{\partial^2 V(x)}{\partial x^2} \Big|_{x=X_n-X_m} (u_n - u_m)^2. \quad (11)$$

In terms of Fourier transforms

$$\delta U = \frac{1}{2} \sum_k m\omega^2(k) |u_k|^2, \quad (12)$$

where u_k is the Fourier transform of the displacement u_n , $-\pi/a < k < \pi/a$, and the phonon spectrum is given by

$$m\omega^2(k) = \sum_{n \neq 0} (1 - e^{-ikX_n}) \frac{\partial^2 V(x)}{\partial x^2} \Big|_{x=X_n} . \quad (13)$$

Approximating $V(x)$ by $V(x) = \gamma/x^2$, the sums in Eq. (13) can be performed exactly. The result is

$$\omega(k) = s |k| \left(1 - \frac{|k|a}{2\pi}\right) , \quad (14)$$

where $s = (\hbar/ma)\pi\sqrt{g/2}$ is the sound velocity. This coincides with the velocity of a compressional sound wave, $s = \sqrt{\partial p / \partial \rho}$, where $p = -\partial E / \partial L$ is the pressure, $\rho = m/a$ is the density and E is given by Eq. (4), if β is approximated as $\beta \simeq \sqrt{2g}$, which is justified for $g \gg 1$.

The ground-state wavefunction in the harmonic approximation Ψ_h is such that

$$|\Psi_h|^2 = \text{const.} \prod_k \exp \left\{ -\frac{m\omega(k)}{\hbar} |u_k|^2 \right\} . \quad (15)$$

Now we show that the exact wavefunction reduces to Ψ_h for $g \gg 1$. In this case $\beta \simeq \sqrt{2g} \gg 1$ and it is justified to apply the harmonic approximation to the fictitious potential $\mathcal{V}(x)$ in Eq. (6). Proceeding exactly as before we find

$$\mathcal{U} \equiv \frac{1}{2} \sum_{i \neq j} \mathcal{V}(x_i - x_j) = \frac{1}{2} \sum_{n \neq m} \mathcal{V}(X_n - X_m) + \delta\mathcal{U} , \quad (16)$$

with

$$\delta\mathcal{U} = \frac{1}{2} \sum_k m\Omega^2(k) |u_k|^2 , \quad (17)$$

where $\Omega(k)$ is the fictitious potential phonon spectrum, given by Eq. (13) with V replaced by \mathcal{V} . Approximating $\mathcal{V}(x)$ by $\mathcal{V}(x) = -\ln|x|$, the sums in Eq. (16) can be performed exactly. The result is

$$m\Omega^2(k) = \frac{\pi}{a} |k| \left(1 - \frac{|k|a}{2\pi}\right) . \quad (18)$$

Substituting Eq. (18) in Eq. (17) and using Eq. (6) it follows that the exact wavefunction coincides with the harmonic approximation one in the limit $g \gg 1$.

B. Structure Function and Correlation Function

The structure function is defined as

$$S(q) = \sum_n e^{iqX_n} \langle e^{iq(u_n - u_0)} \rangle , \quad (19)$$

where $\langle \rangle$ denotes the ground-state average.

In the harmonic approximation, where the ground-state wavefunction, Eq. (15), is gaussian,

$$\langle e^{iq(u_n - u_0)} \rangle_h = e^{-\frac{1}{2} \langle |q(u_n - u_0)|^2 \rangle_h} , \quad (20)$$

where

$$\langle |q(u_n - u_0)|^2 \rangle_h = \frac{2q^2}{N} \sum_k \langle |u_k|^2 \rangle_h (1 - \cos kX_n) . \quad (21)$$

It follows from Eq. (15) that,

$$\langle |u_k|^2 \rangle_h = \frac{\hbar}{2m\omega(k)} . \quad (22)$$

Using Eq. (14) we find that

$$\langle |q(u_n - u_0)|^2 \rangle_h = 2\eta^h(q) [\ln(2\pi n) + C - Ci(2\pi n)] , \quad (23)$$

where $C = 0.577$ is Euler's constant, $Ci(x)$ is the cosine-integral function, defined as in Ref. [8], and

$$\eta^h(q) = \frac{q^2 a^2}{\pi^2 \sqrt{2g}} . \quad (24)$$

The above result shows that the displacement correlation function, Eq. (20), decays algebraically with distance, that is

$$\langle e^{iq(u_n - u_0)} \rangle_h = F(n) |n|^{-\eta^h(q)} , \quad (25)$$

where $F(n) = \exp -\eta^h(q) [\ln(2\pi) + C - Ci(2\pi n)]$ is an oscillating function of n . From this result it follows, using Eq. (19), that for q near a reciprocal lattice vector $G_p = (2\pi/a)p$ (p = integer) such that $\eta^h(G_p) < 1$, $S(q)$, in the harmonic approximation, has power law singularities, namely

$$S_h(G_p + K) = \text{const.} |K|^{\eta^h(G_p) - 1} , \quad (26)$$

where $K \ll G_p$.

IV. NUMERICAL CALCULATIONS

In this Section we discuss the Monte Carlo method used to calculate ground-state averages of the type given by Eq. (8) and report the results obtained from it.

A. Monte Carlo Method

Our numerical method consists of calculating averages from Eq. (8) using the traditional Monte Carlo method [6]. For a given L we first simulate the largest β value. In this case the initial configuration is an ordered chain of N particles. New configurations are generated by displacing particles, one at a time, in a sequential way along the chain. The Metropolis algorithm is used to accept or reject configurations. For smaller β values we use as the initial configuration the last one generated in the previous run. Typical runs consist of 1000 MC steps per particle to equilibrate and 5000 MC steps per particle to calculate averages. We simulate systems with fixed $a = 50$ and with N ranging from $N = 100$ to $N = 1000$. The use of a single a value is justified because in the CS model the correlation functions depend only on a through x/a or qa .

Our aim is to investigate by this method the behavior of the displacement correlation functions $\langle e^{iG_p(u_n - u_0)} \rangle$, for $n \gg 1$. There are three possibilities.

- i- QLRPO: $\langle e^{iG_p(u_n - u_0)} \rangle \rightarrow |n|^{-\eta(G_p)}$.
- ii- True long-range positional order: $\langle e^{iG_p(u_n - u_0)} \rangle \rightarrow \text{const.}$
- iii- Exponential decay: $\langle e^{iG_p(u_n - u_0)} \rangle \rightarrow e^{-n/\xi}$.

For $L = \infty$, both true long-range positional order and QLRPO with $\eta(G_p) < 1$ lead to divergencies in $S(G_p)$. For finite L these singularities are replaced by peaks of finite height. In order to determine whether peaks correspond to singularities we perform the following finite size scaling analysis of the data. For a given β we compute $S(G_p)$ for several values of L . Next we fit this data to

$$S(G_p) = \text{const.} |L|^{1-\eta(G_p)}. \quad (27)$$

This dependence on L arises as follows. If the system has QLRPO with $\eta(G_p) < 1$, Eq. (19) predicts that $S(G_p)$ diverges with L according to Eq. (27). If true long-range positional order is present Eq. (19) predicts $S(G_p) \sim L$ and Eq. (27) gives $\eta(G_p) = 0$.

This finite size scaling method cannot distinguish between QLRPO with $\eta(G_p) \geq 1$ and exponential decay. In both cases $S(G_p)$ becomes L -independent, because there is no singularity. Thus, fitting the $S(G_p)$ data to Eq. (27) results in $\eta(G_p) = 1$.

In order to distinguish between these two possibilities we compute, for a given L , $\langle e^{iG_p(u_n - u_0)} \rangle$ and study its behavior as a function of n .

B. Results

We find that $S(q)$, has peaks at reciprocal lattice vectors $q = G_p = 2\pi p/a$, with $p = 0, \pm 1, \dots, \pm p_{\max}$. The

value of p_{\max} depends on β and increases with increasing β . In Fig. 1 we show $S(q)$ for $\beta = 18$, where two peaks are observed at $q = G_1$ and $q = G_2$. As β decreases, the peak at $q = G_2$ is no longer observed for $\beta \lesssim 12$. Below this value there is only one peak at $q = G_1$. This peak disappears for a value of $\beta < 4$.

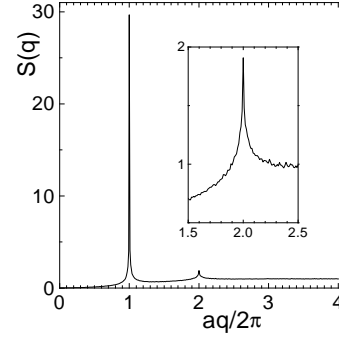


FIG. 1. Structure function as a function of q for $\beta = 18$, $L = 5000$ and $N = 100$. Inset shows in detail the peak at $q = G_2$.

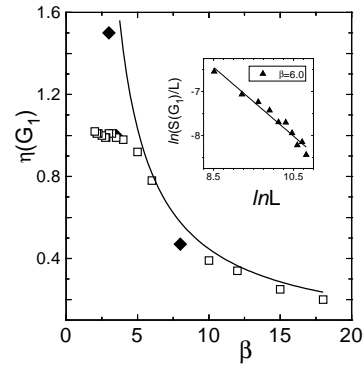


FIG. 2. Exponent $\eta(G_1)$ as a function of β . Solid squares: finite size scaling of MC data. Inset: typical plot used to obtain $\eta(G_1)$. Continuous line: harmonic approximation. Open squares: $\eta(G_1)$ obtained from the decay of $\langle e^{iG_1(u_n - u_0)} \rangle$ with n for $L = 5000$ and $N = 100$.

To identify whether or not these peaks correspond to singularities we carry out the finite size scaling analysis described above. This is illustrated in Fig. 2. We find that the dependence of the exponent $\eta(G_1)$ on β is that shown in Fig. 2.

We interpret the β -dependence of $\eta(G_1)$ shown in Fig. 2 as follows. The numerical estimates for $\eta(G_1)$ are in good agreement with harmonic approximation predictions for $\beta > 5$. For $\beta = 4$ Sutherland has obtained $S(q)$ exactly. A logarithmic singularity occurs at $q = G_1$, that is $S(q) \propto \ln |q - G_1|$, rather than an algebraic one. We find that at $\beta = 4$, $\eta(G_1) = 0.98$ (Fig. 3). We believe that this is consistent with Sutherland's exact result. The reason is that the logarithm divergence leads to $S(G_1) \sim \ln L$ in a finite system. It is well known that if this function is fitted to Eq. (27) on a log-log plot it leads to an exponent equal to zero [9], which corresponds to $\eta(G_1) = 1$. Our estimate is, within the accuracy of our simulations, consistent with that. For $\beta = 4$ our numerical results for $S(q)$ agree well with the exact ones, as shown in Fig. 3. For $2 < \beta < 4$ our results fitted to Eq. (27) give $\eta(G_1) \sim 1$. As discussed in Sec. IV A, this only indicates that $S(q)$ has no singularity at $q = G_1$, which is consistent with either QLRPO with $\eta(G_1) > 1$ or with exponential decay.

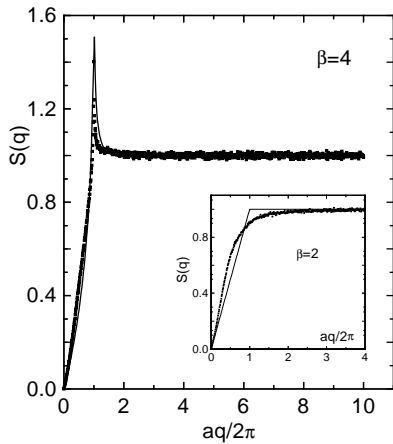


FIG. 3. Structure function as a function of q . Comparison between MC results for $L = 5000$ and $N = 100$ (solid squares) and exact results (continuous line) for $\beta = 4$ and $\beta = 2$ (inset).

In order to investigate which of these possibilities occur in the region $2 < \beta < 4$ we compute the displacement correlation function, $\langle e^{iG_1(u_n - u_0)} \rangle$, as a function of n , for a given L . We find that, as a result of strong quantum fluctuations, very long MC runs are necessary in order to obtain reasonably accurate correlation func-

tions. Our results for $\beta = 8.0, 3.5$ and 3 , shown in Figs. 2 and 4 require 10^6 MC steps. For $\beta = 3$ and 3.5 we find that $\langle e^{iG_1(u_n - u_0)} \rangle$ decays with n slower than the harmonic approximation predicts and even becomes negative at $n = 1$, as shown in Fig. 4. We also find that for $\beta = 3.0$ and 3.5 it decays as $n^{-1.5}$ and $n^{-1.0}$, respectively. We interpret this as evidence that QLRPO with $\eta(G_1) > 1$ is also present for $2 < \beta < 4$. Thus, there is no phase with exponential decay of positional correlations in the CS model.

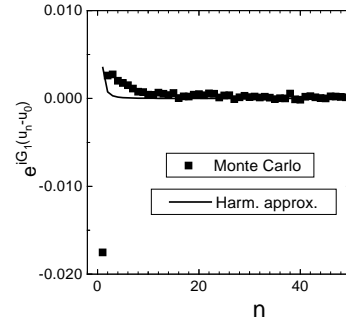


FIG. 4. Displacement correlation function as a function of n . Data points: MC data for $L = 5000$ and $N = 100$. Continuous line: harmonic approximation.

For $\beta = 2$ Sutherland has also obtained $S(q)$ exactly. In Fig. 3 we show that our results for $\beta = 2$ agree reasonably well with the exact ones. We attribute the rounding off near $q = G_1$ to finite size effect.

We also calculate the exponent $\eta(G_2)$ by the finite size scaling method for a few β values. We find that, in the region where $S(G_2)$ has a singularity, $\eta(G_2) < 1$ and its estimated value is in close agreement with the harmonic approximation one, $\eta^h(G_2)$. This approximation predicts that $\eta^h(G_2) \leq 1$ for $\beta \geq 17$ ($g \geq 128$). We did not attempt to estimate $\eta(G_2)$ outside the region where $S(G_2)$ has no singularity from the displacement correlation function $\langle e^{iG_2(u_n - u_0)} \rangle$.

V. DISCUSSION

The main conclusion of our numerical study is that the ground-state of the CS model for $g > 0$ or $2 \leq \beta < \infty$ has QLRPO characterized by an exponent that varies continuously with β . Our results also reveal that, as far as positional correlations are concerned, three distinct param-

ter regions can be identified: i) A semiclassical region for $\beta > 5$ ($g > 7.5$) in which $\eta(G_1) \simeq \eta^h(G_1)$. As discussed in Sec.III this is expected for large g where the exact CS ground-state wavefunction coincides with the harmonic approximation one. Our results show that for $g > 7.5$ the CS model behaves semiclassically. ii) A region of moderate quantum fluctuations, for $4 < \beta < 5$ ($4 < g < 7.5$), where $S(q)$ has one algebraic singularity at $q = G_1$ with exponent $\eta(G_1) < 1$. In this region anharmonic quantum fluctuations are important, leading to $\eta(G_1) < \eta^h(G_1)$. iii) A strong quantum fluctuations region, for $2 < \beta < 4$, where $S(q)$ has no singularities, but QLRPO exists with $\eta(G_1) > 1$. As a result of strong quantum fluctuations the displacement correlation function at short distances becomes negative, in contrast to the harmonic approximation that predicts positive value for all n . At large distances it decays algebraically with distance but with an exponent such that $\eta(G_1) \ll \eta^h(G_1)$.

It is instructive to estimate the amplitude of quantum fluctuations in the boson positions using a 'cage model' [10]. This model considers a single particle moving in the potential well produced by the other bosons, assumed fixed in their classical positions. In this case we find that, in the harmonic approximation, the boson zero-point mean-square displacement from its classical equilibrium position is $\langle u^2 \rangle / a^2 = 0.55 / \sqrt{g}$. According to the discussion above, the semiclassical region corresponds to $g > 7.5$, for which $\sqrt{\langle u^2 \rangle} / a^2 < 0.45$. We propose that this last result be adopted as a kind of 'Lindemann criterion' [7] to estimate the range of validity of the harmonic approximation for 1D interacting boson models in general.

An important conclusion that can be drawn from these results is that, as far as QLRPO is concerned, the harmonic approximation gives a correct qualitative picture for the behavior of the CS model ground-state. The differences that we find between the exact result and this approximation, although large for $\beta < 5$, are quantitative rather than qualitative. The fact that this conclusion is reached in an exactly soluble model suggests that for other 1D boson models the harmonic approximation predictions for QLRPO are correct, at least qualitatively, well beyond the semiclassical region, as long as a phase transition does not take place.

One possibility that cannot be studied by our method is the existence of a superfluid phase. We believe that this phase can be ruled out in the CS model because the bosons are impenetrable. In 1D this excludes the possibility of a superfluid phase. The reason is that if one constructs the path-integral representation for the

ground-state partition function, the superfluid density is related to the boson world lines winding number fluctuations [11]. In 1D these fluctuations require that the boson world lines cross each other, which is not possible if the bosons are impenetrable. From this and from our numerical results we conclude that the ground-state of the repulsive CS model has only one normal phase with QLRPO. We believe that a large class of 1D models for impenetrable bosons interacting through a repulsive potential falling off so fast with distance to exclude true long-range positional order has a similar ground-state phase diagram.

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- [1] N.D. Mermin, Phys. Rev. **176**, 250 (1968).
- [2] D.R. Nelson *Phase Transitions and Critical Phenomena Vol. 7*, edited C. Domb and M.S. Green, (Academic Press, London, 1983)
- [3] J. Tobochnik and G. V. Chester, Phys. Rev. B **25**, 6778 (1982).
- [4] B. Sutherland, Phys. Rev. A **4**, 2019 (1971); J. Math. Phys. **12**, 246 (1971).
- [5] V. Ya. Krivnov and A.A. Ovchinnikov, Zh. Eksp. Teor. Fiz. **82**, 271 (1982) [Sov. Phys. JETP **55**, 162 (1982)]; A. V. Zabrodin and A.A. Ovchinnikov, Zh. Eksp. Teor. Fiz. **90**, 2260 (1986) [Sov. Phys. JETP **63**, 1326 (1986)]; P.J. Forrester, J. Stat. Phys. **72**, 39 (1993); D. Sen and R.K. Bhaduri, preprint cond-mat 9705279
- [6] See, e.g., *Monte Carlo Methods in Statistical Physics*, edited by K. Binder (Springer, Berlin, 1979).
- [7] D. Pines *Elementary Excitations in Solids* (Benjamin, N.Y., 1963).
- [8] M. Abramowitz I.A. Segun, *Handbook of Mathematical Functions* (Dover, N.Y., 1965).
- [9] H.E. Stanley *Introduction to Phase Transitions* (Oxford, Oxford, 1971).
- [10] E. Frey, D.R. Nelson and D.S. Fisher, Phys. Rev. B **49**, 9723 (1994).
- [11] D.M. Ceperley and E.L. Pollock, Phys. Rev. B **39**, 2084 (1989).